3 Indices and Cube root

Let's recall.

In earlier standards, we have learnt about Indices and laws of indices.

- The product $2 \times 2 \times 2 \times 2 \times 2$, can be expressed as 2^5, in which 2 is the base, 5 is the index and 2^5 is the index form of the number.
- Laws of indices : If m and n are integers, then

(i) $a^m \times a^n = a^{m+n}$ (ii) $a^m \div a^n = a^{m-n}$ (iii) $(a \times b)^m = a^m \times b^m$ (iv) $a^0 = 1$

(v) $a^{-m} = \frac{1}{a^m}$ (vi) $(a^m)^n = a^{mn}$ (vii) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ (viii) $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

- Using laws of indices, write proper numbers in the following boxes.

(i) $3^5 \times 3^2 = 3^{\square}$ (ii) $3^7 \div 3^9 = 3^{\square}$ (iii) $(3^4)^5 = 3^{\square}$

(iv) $5^{-3} = \frac{1}{5^{\square}}$ (v) $5^0 = \square$ (vi) $5^1 = \square$

(vii) $(5 \times 7)^2 = 5^{\square} \times 7^{\square}$ (viii) $\left(\frac{5}{7}\right)^3 = \frac{\square^3}{\square^3}$ (ix) $\left(\frac{5}{7}\right)^{-3} = \left(\frac{\square}{\square}\right)^3$

Let's learn.

Meaning of numbers with rational indices

(I) Meaning of the numbers when the index is a rational number of the form $\frac{1}{n}$.

Let us see the meaning of indices in the form of rational numbers such as $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, ..., \frac{1}{n}$.

To show the square of a number, the index is written as 2 and to show the square root of a number, the index is written as $\frac{1}{2}$.

For example, square root of 25, is written as $\sqrt{25}$ using the radical sign '$\sqrt{\ }$'.

Using index, it is expressed as $25^{\frac{1}{2}}$. $\therefore \sqrt{25} = 25^{\frac{1}{2}}$.

In general, square of a can be written as a^2 and square root of a is written as $\sqrt[2]{a}$ or $\sqrt{a}$ or $a^{\frac{1}{2}}$.

Similarly, cube of a is written as a^3 and cube root of a is written as $\sqrt[3]{a}$ or $a^{\frac{1}{3}}$.

For example, $4^3 = 4 \times 4 \times 4 = 64$.

$\therefore$ cube root of 64 can be written as $\sqrt[3]{64}$ or $(64)^{\frac{1}{3}}$. Note that, $64^{\frac{1}{3}} = 4$

$3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 243$. That is 5^{th} power of 3 is 243.

Conversely, 5^{th} root of 243 is expressed as $(243)^{\frac{1}{5}}$ or $\sqrt[5]{243}$. Hence, $(243)^{\frac{1}{5}} = 3$

In genral n^{th} root of a is expressed as $a^{\frac{1}{n}}$.

For example, (i) $128^{\frac{1}{7}} = 7^{th}$ root of 128, (ii) $900^{\frac{1}{12}}$ = 12th root of 900, etc.

Note that, If $10^{\frac{1}{5}} = x$ then $x^5 = 10$.

Practice Set 3.1

1. Express the following numbers in index form.

(1) Fifth root of 13 (2) Sixth root of 9 (3) Square root of 256

(4) Cube root of 17 (5) Eighth root of 100 (6) Seventh root of 30

2. Write in the form 'n^{th} root of a' in each of the following numbers.

(1) $(81)^{\frac{1}{4}}$ (2) $49^{\frac{1}{2}}$ (3) $(15)^{\frac{1}{5}}$ (4) $(512)^{\frac{1}{9}}$ (5) $100^{\frac{1}{19}}$ (6) $(6)^{\frac{1}{7}}$

(II) The meaning of numbers, having index in the rational form $\frac{m}{n}$.

We know that $8^2 = 64$,

Cube root at 64 is = $(64)^{\frac{1}{3}} = (8^2)^{\frac{1}{3}} = 4$

$\therefore$ cube root of square of 8 is 4 .... (I)

Similarly, cube root of 8 = $8^{\frac{1}{3}} = 2$

$\therefore$ square of cube root of 8 is $\left(8^{\frac{1}{3}}\right)^2 = 2^2 = 4$ ....(II)

From (I) and (II)

cube root of square of 8 = square of cube root of 8. Using indices, $(8^2)^{\frac{1}{3}} = \left(8^{\frac{1}{3}}\right)^2$.

The rules for rational indices are the same as those for integral indices

$\therefore$ using the rule $\boldsymbol{(a^m)^n = a^{mn}}$, we get $(8^2)^{\frac{1}{3}} = \left(8^{\frac{1}{3}}\right)^2 = 8^{\frac{2}{3}}$.

From this we get two meanings of the number $8^{\frac{2}{3}}$.

(i) $8^{\frac{2}{3}} = (8^2)^{\frac{1}{3}}$ i. e. cube root of square of 8.

(ii) $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2$ i. e. square of cube root of 8.

Similarly, $27^{\frac{4}{5}} = \left(27^4\right)^{\frac{1}{5}}$ means 'fifth root of fourth power of 27',

and $27^{\frac{4}{5}} = \left(27^{\frac{1}{5}}\right)^4$ means 'fourth power of fifth root of 27'.

Generally we can express two meanings of the number $a^{\frac{m}{n}}$.

$a^{\frac{m}{n}} = \left(a^m\right)^{\frac{1}{n}}$ means 'n^{th} root of m^{th} power of a'.

$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$ means 'm^{th} power of n^{th} root of a'.

Practice Set 3.2

1. Complete the following table.

Sr. No.	Number	Power of the root	Root of the power
(1)	$(225)^{\frac{3}{2}}$	Cube of square root of 225	Square root of cube of 225
(2)	$(45)^{\frac{4}{5}}$		
(3)	$(81)^{\frac{6}{7}}$		
(4)	$(100)^{\frac{4}{10}}$		
(5)	$(21)^{\frac{3}{7}}$		

2. Write the following numbers in the form of rational indices.

(1) Square root of 5th power of 121. (2) Cube of 4th root of 324

(3) 5th root of square of 264 (4) Cube of cube root of 3

Let's recall.

- $4 \times 4 = 16$ implies $4^2 = 16$, also $(-4)^2 = 16$ which indicates that the number 16 has two square roots ; one positive and the other negative. Conventionally, positive root of 16 is shown as $\sqrt{16}$ and negative root of 16 is shown as $-\sqrt{16}$. Hence $\sqrt{16} = 4$ and $-\sqrt{16} = -4$.
- Every positive number has two square roots.
- Square root of zero is zero.

Let's learn.

Cube and Cube Root

If a number is written 3 times and multiplied, then the product is called the cube of the number. For example, $6 \times 6 \times 6 = 6^3 = 216$. Hence 216 is the cube of 6.

To find the cube of rational number.

Ex. (1) Find the cube of 17.

$17^3 = 17 \times 17 \times 17$

$= 4913$

Ex. (2) Find the cube of (-6).

$(-6)^3 = (-6) \times (-6) \times (-6)$

$= -216$

Ex. (3) Find the cube of $\left(-\frac{2}{5}\right)$.

$\left(-\frac{2}{5}\right)^3 = \left(-\frac{2}{5}\right) \times \left(-\frac{2}{5}\right) \times \left(-\frac{2}{5}\right)$

$= -\frac{8}{125}$

Ex. (4) Find the cube of (1.2).

$(1.2)^3 = 1.2 \times 1.2 \times 1.2$

$= 1.728$

Ex. (5) Find the cube of (0.02).

$(0.02)^3 = 0.02 \times 0.02 \times 0.02$

$= 0.000008$

Use your brain power.

In Ex. (1) 17 is a positive number. The cube of 17, which is 4913, is also a positive number.

In Ex. (2) cube of - 6 is -216. Take some more positive and negative numbers and obtain their cubes. Find the relation between the sign of a number and the sign of its cube.

In Ex. (4) and (5), observe the number of decimal places in the number and number of decimal places in the cube of the number. Is there any relation between the two ?

To find the cube root

We know, how to find the square root of a number by factorisation method. Using the same method, we can find the cube root.

Ex. (1) Find the cube root of 216.

Solution : First find the prime factor of 216. $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Each of the factors 3 and 2, appears thrice. So let us group them as given below,

$216 = (3 \times 2) \times (3 \times 2) \times (3 \times 2) = (3 \times 2)^3 = 6^3$

$\therefore \sqrt[3]{216} = 6$ that is $(216)^{\frac{1}{3}} = 6$

Ex. (2) Find the cube root of -1331.

Solution : To find the cube root of -1331, let us factorise 1331 first.

$1331 = 11 \times 11 \times 11 = 11^3$

$-1331 = (-11) \times (-11) \times (-11)$
$= (-11)^3$

$\therefore \sqrt[3]{-1331} = -11$

Ex. (4) Find $\sqrt[3]{0.125}$.

Solution : $\sqrt[3]{0.125} = \sqrt[3]{\frac{125}{1000}}$

$= \frac{\sqrt[3]{125}}{\sqrt[3]{1000}} \quad \cdots \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

$= \frac{\sqrt[3]{5^3}}{\sqrt[3]{10^3}}$

$= \frac{5}{10} = 0.5 \quad \cdots \left(a^m\right)^{\frac{1}{m}} = a$

Ex. (3) Find the cube root of 1728.

Solution : $1728 = 8 \times 216 = 2 \times 2 \times 2 \times 6 \times 6 \times 6$

$\therefore 1728 = 2^3 \times 6^3 = (2 \times 6)^3$ .... $a^m \times b^m = (a \times b)^m$

$\sqrt[3]{1728} = 2 \times 6 = 12$ (Note that, cube root of -1728 is -12.)

Practice Set 3.3

1. Find the cube roots of the following numbers.

(1) 8000 (2) 729 (3) 343 (4) -512 (5) -2744 (6) 32768

2. Simplify : (1) $\sqrt[3]{\frac{27}{125}}$ (2) $\sqrt[3]{\frac{16}{54}}$

3. If $\sqrt[3]{729} = 9$ then $\sqrt[3]{0.000729} = ?$

Answers

Practice Set 3.1 (1) $13^{\frac{1}{5}}$ (2) $9^{\frac{1}{6}}$ (3) $256^{\frac{1}{2}}$ (4) $17^{\frac{1}{3}}$ (5) $100^{\frac{1}{8}}$ (6) $30^{\frac{1}{7}}$

2. (1) Fourth root of 81. (2) Square root of 49 (3) Fifth root of 15 (4) Ninth root of 512 (5) Nineteenth root of 100 (6) Seventh root of 6

Practice Set 3.2 **1.** (2) 4^{th} power of 5^{th} root of 45 ; 5^{th} root of 4^{th} power of 45.
(3) 6^{th} power of 7^{th} root of 81 ; 7^{th} root of 6^{th} power of 81.
(4) 4^{th} power of 10^{th} root of 100 ; 10^{th} root of 4^{th} power of 100.
(5) 3^{rd} power of 7^{th} root of 21 ; 7^{th} root of 3^{rd} power of 21.

2. (1) $(121)^{\frac{5}{2}}$ (2) $(324)^{\frac{3}{4}}$ (3) $(264)^{\frac{2}{5}}$ (4) $3^{\frac{3}{3}}$

Practice Set 3.3 **1.** (1) 20 (2) 9 (3) 7 (4) -8 (5) -14 (6) 32 **2.** (1) $\frac{3}{5}$ (2) $\frac{2}{3}$ **3.** 0.09